



FURTHER MATHEMATICS

9795/02

Paper 2 Further Application of Mathematics

May/June 2018

MARK SCHEME

Maximum Mark: 120

Published

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Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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This document consists of **15** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Question	Answer	Marks	Partial Marks
1(i)	$B(200, 0.2) \approx N(40, 32)$	M1	$N(40, \dots)$
	$\Phi\left(\frac{30.5 - 40}{\sqrt{32}}\right) = \Phi(-1.679)$	A1	variance 32 soi
		M1	standardising
	$= 0.0466$	A1	answer in [0.046, 0.047]
1(ii)	$Y \sim B(200, 0.02) \approx \text{Po}(\dots$	M1	Poisson, any λ
	$\dots 4)$	A1	λ (or np) = 4
	$P(Y \leq 3) = 0.433$	A1	answer in [0.433, 0.434]
2(i)	Breaks in transmission must occur independently of one another	B1	independent, in context
	and at constant average rate or constant expected rate	B1	'constant average rate' or 'constant expected rate' or 'mean \propto time int'
2(ii)	$X \sim \text{Po}(7.5)$	M1	Po(7.5) stated or implied
	$P(6 \leq X \leq 10) = P(X \leq 10) - P(X \leq 5)$	M1	Po, $P(\leq 10) - P(\leq 5)$ or 6)
	$= 0.8622 - 0.2414 = 0.621$	A1	awrt 0.621
2(iii)	$P(0) = e^{-\lambda}$	M1	correct use of $P(0)$ formula
	$e^{-1.3T} = 0.6$	A1	e^{-kT} or $e^{-\lambda} = 0.6$ (provided not clearly incorrect Poisson e.g. Po(8))
	$-1.3 T = \ln(0.6) [= -0.5108]$	M1	dep on first M mark for taking logarithms
	$T (= 0.393 \text{ min}) = 23.6 \text{ s}$	A1	23.6 or 24 s or better
3(i)	$M'(t) = 6(1 - 2t)^{-4}$	M1	attempt to differentiate $M(t)$
	$M'(0) = 6$ so $E(X) = 6$	A1	6 correctly obtained
	$M''(t) = 48(1 - 2t)^{-5}$ [$M''(0) = 48$]	M1	attempting $M''(t)$
	Therefore $\text{Var}(X) = 48 - 6^2$	M1	$M''(0) - [M'(0)]^2$ used
	$= 12$	A1	12 correctly obtained

Question	Answer	Marks	Partial Marks
3(i)	Alternative $M_X(t) = 1 + 6t + \dots$	M1	attempt to expand: 1st 2 terms
	$E(X) = 6$	A1	6 correctly (soi) obtained
	$M_X(t) = "1 + 6t" + 24t^2 + \dots$	M1	3rd term and using 2!
	Therefore $\text{Var}(X) = 2 \times 24 - 6^2$	M1	$E(X^2) - [E(X)]^2$ used
	$= 12$	A1	12 correctly obtained
3(ii)	$Y = X_1 + X_2; M_Y(t) = (1 - 2t)^{-6}$	M1	$[M_X(t)]^2$
		M1	attempt to find $M_Y'''(t)$
	$M_Y''' = 2688(1 - 2t)^{-9}$	A1	correct expression
	Therefore $E(Y^3) = M_Y'''(0) = 2688$	A1	answer 2688 only
	Alternative 1 $Y = X_1 + X_2; M_Y(t) = (1 - 2t)^{-6}$	M1	$[M_X(t)]^2$
		M1	using coefficient of t^3
	$\frac{1}{3!} E(Y^3) = (-2)^3 \frac{(-6)(-7)(-8)}{1 \times 2 \times 3}$	A1	correct expression inc. 3!
	Therefore $E(Y^3) = 6 \times 7 \times 8^2 = 2688$	A1	answer 2688 only
	Alternative 2 $M_X(t) = "1 + 6t + 24t^2 + " + 80 \text{ soi}$	M1	genuine attempt at next term
		M1	Dep Attempt to expand and correct use of expectation algebra
	$E((X_1 + X_2)^3) = 2E(X^3) + 6E(X^2)E(X) \text{ oe}$	A1	
	$2 \times 480 + 6 \times 48 \times 6 = 2688$	A1	answer 2688 only
4(i)	$f(x) = \frac{3}{8}x^2$	M1	attempt to differentiate
	$\int_0^2 \frac{3}{8}x^2 \cdot x \, dx = \left[\frac{3}{32}x^4 \right]_0^2$	M1	$\int xf(x) \, dx$ <u>and</u> limits 0, 2
	$= 1\frac{1}{2} \text{ oe}$	A1	exact answer

Question	Answer	Marks	Partial Marks
4(ii)	$Y = \frac{1}{X^2} : P(Y \leq y) = P\left(\frac{1}{X^2} \leq y\right)$	M1	appropriate probability statement
	$= P\left(X \geq \frac{1}{\sqrt{y}}\right)$	M1	using inverse function correctly somewhere
	$= 1 - F\left(\frac{1}{\sqrt{y}}\right)$ or $1 - \frac{1}{8}y^{-\frac{3}{2}}$	M1	1 – F(inverse function)
		M1	differentiating their F(y) (wrt correct variable)
	$\Rightarrow f(y) = \frac{3}{16}y^{-\frac{5}{2}}$	A1	correct formula for f(y), cwo
	for $y \geq \frac{1}{4}$ (must be correct variable)	B1	correct range for non-zero f
	Alternative		
	$f_Y(y) = f_X(g^{-1}(y)) \left \frac{d}{dx}(g^{-1}(y)) \right $	B1	statement
	$g^{-1}(y) = \frac{1}{\sqrt{y}} = y^{-\frac{1}{2}}$	B1	correct inverse function (condone \pm)
	$f_X(g^{-1}(y)) = \frac{3}{8}(y^{-\frac{1}{2}})^2 = \frac{3}{8}y^{-1}$	B1	
	$\frac{d}{dx}(g^{-1}(y)) = -\frac{1}{2}y^{-\frac{3}{2}}$	B1	
	$f_Y(y) = \frac{3}{8}y^{-1} \left -\frac{1}{2}y^{-\frac{3}{2}} \right = \frac{3}{16}y^{-\frac{5}{2}}$	B1	
for $y \geq \frac{1}{4}$ (must be correct variable)	B1	correct range for non-zero f	

Question	Answer	Marks	Partial Marks
5(i)	$\bar{t}_B = 17, \bar{t}_G = 22.29$	B1	both sample means seen
	$S_B^2 = 64 (S_B = 8), S_G^2 = 12.35 (S_G = 3.514)$	M1	method for finding both Ss (or $S_{BB} (=768)$ and $S_{GG} (= 1210/7)$)
	$\hat{\sigma}^2 = \frac{12S_B^2 + 14S_G^2}{12 + 14 - 2} = 39.20 (\hat{\sigma} = 6.261)$	M1	proper attempt at pooled variance (no working and wrong M0)
	$\bar{t}_G - \bar{t}_B \pm 2.064\sqrt{\hat{\sigma}^2(\frac{1}{12} + \frac{1}{14})}$	A1	correct variance estimate
		M1	CI with t or z and $\frac{1}{12} + \frac{1}{14}$ soi ($5.2857 \pm (2.064 \times 2.463)$)
		B1	$t = 2.064$ used in a CI
	$= (0.202, 10.4)$ (3sf or better (cwo))	A2	A1 for each end-point, allow if both negative (i.e. $(-10.4, -0.202)$)
	Assume distributions of boys' and girls' times normally distributed with common variance oe	B1	
5(ii)	No as 0 is not in the confidence interval	B1	FT
6(i)	$P(X = r) = \frac{1}{N}$	B1	PDF $\frac{1}{N}$ stated or implied
	$E(X) = \sum_{x=1}^N \frac{1}{N} x = \frac{N(N+1)}{2N}$	M1	using $\sum xf(x)$ and $\frac{1}{2}N(N+1)$
	$= \frac{1}{2}(N+1)$ AG	A1	correctly obtaining AG
	$E(X^2) = \sum_{x=1}^N \frac{1}{N} x^2 = \frac{(N+1)(2N+1)}{6}$	M1	using $\sum x^2 f(x)$ and $\frac{1}{6}N(N+1)(2N+1)$
	$\text{Var}(X) = \frac{(N+1)(2N+1)}{6} - \left(\frac{N+1}{2}\right)^2$	M1	subtracting their $[E(X)]^2$
	$= \frac{1}{12}(N+1)[2(2N+1) - 3(N+1)]$ $= \frac{1}{12}(N^2 - 1)$ AG	A1	correctly obtaining AG

Question	Answer	Marks	Partial Marks
6(i)	Alternative PGF $G(t) = (t + t^2 + \dots + t^N) / N$ so $G'(t) = (1 + 2t + \dots + Nt^{N-1}) / N$	M1	Correct PGF and differentiation
	$E(X) = G'(1) = \sum_{x=1}^N \frac{1}{N} x = \frac{N(N+1)}{2N}$	M1	equating and using $\sum xf(x)$ and $\frac{1}{2}N(N+1)$
	$= \frac{1}{2}(N+1)$ AG	A1	correctly obtaining AG
	$\text{Var}(X) = G''(1) + G'(1) - [G'(1)]^2$ $G''(t) = (2 + 3 \times 2t + 4 \times 3t + \dots + N(N-1)t^{N-2}) / N$	M1	for correct formula <u>and</u> $G''(t)$
	$\text{Var}(X) = \frac{1}{N} \sum_{x=1}^N x(x-1) + \frac{1}{N} \sum_{x=1}^N x - \left(\frac{N+1}{2}\right)^2$ $\text{Var}(X) = \frac{1}{N} \sum_{x=1}^N x^2 - \frac{1}{4}(N+1)^2$ $\text{Var}(X) = \frac{(N+1)(2N+1)}{6} - \frac{1}{4}(N+1)^2$	M1	dep for simplifying <u>and</u> using $\frac{1}{6}N(N+1)(2N+1)$
	$= \frac{1}{12}(N+1)[2(2N+1) - 3(N+1)]$ $= \frac{1}{12}(N^2 - 1)$ AG	A1	correctly obtaining AG
6(ii)	$E(A) = \frac{1}{2}(N+1)$	M1	correct statement of $E(A)$
	Therefore $E_1 = 2A - 1$ (is an unbiased estimator)	A1	correct UE
	$\text{Var}(A) = \frac{N^2 - 1}{12 \times 40}$ aef	B1	correct variance of A soi
	$E_1 \sim \text{Normal}(N, \text{any } \sigma^2)$ or (any $\mu, \frac{1}{120}(N^2 - 1)$)	M1	Normal stated, correct μ or σ^2
	$\sigma^2 = \text{Var}(E_1) = 4\text{Var}(A) = \frac{1}{120}(N^2 - 1)$	A1	FT for correct mean and variance of UE in (ii) only

Question	Answer	Marks	Partial Marks
6(iii)	$E_2 = \frac{27}{40} B$	B1	but not X or $N = \frac{27}{40} B$
	$\text{Var}(E_2) = \left(\frac{27}{40}\right)^2 \times \alpha N^2$	B1	FT $q^2 \alpha N^2$ from $E_2 = qB + c$ only
	$\left(\frac{27}{40}\right)^2 \times \alpha N^2 > \frac{1}{120}(N^2 - 1) \Rightarrow \frac{2187}{40} \alpha > 1 - \frac{1}{N^2}$ oe	M1	inequality <u>and</u> correct useful simplification (e.g. to $a\alpha > b\left(1 - \frac{1}{N^2}\right)$)
	True for all N iff $\frac{2187}{40} \alpha \geq 1$	B1	convincingly dealing with all N . Condone >
	$\alpha \geq \frac{40}{2187}$ [= awrt 0. 0183]	A1	correct final inequality, even if decimals used. Condone >
7(i)	$\frac{1}{2} \cdot 800(20^2 - 10^2) + 800g \times 16$	M1	use of both KE and GPE
		A1	correct formulation
	= 248 kJ from correct working	A1	
	Alternative $20^2 = 10^2 + 2a \times \frac{16}{\sin \theta}$ and $D - 800g \sin \theta = 800a$	M1	correct use of <u>both</u> $v^2 = u^2 + 2as$ <u>and</u> NII with correct distance
	Eliminating a leads to $WD = D \times \frac{16}{\sin \theta} = 248000\text{J}$	A1	correct answer
	Assuming that a is constant	A1	giving assumption
7(ii)		M1	$\frac{189000}{v}$ soi
	$\frac{189000}{v} - 7v^2 = 0$	A1	correct equation
	$v = \sqrt[3]{27000} = 30 \text{ (ms}^{-1}\text{)}$	A1	for $v = 30$
8(i)	$mg = \frac{\lambda}{l} e$	M1	$mg = \frac{\lambda}{l} e$ used
	$\Rightarrow e = 0.1$	A1	

Question	Answer	Marks	Partial Marks
8(ii)	$\frac{\lambda}{2l}(x-0.2)^2$	M1	using formula for EPE
	$\frac{\lambda}{2l}(x-0.2)^2 = mgx$	M1	equating EPE and GPE
	$20(x-0.2)^2 = 4x$	A1	correct equation
	$25x^2 - 15x + 1 = 0$ oe	A1	correct simplified quadratic
	$x = \frac{3+\sqrt{5}}{10} = 0.524$ (or $x \geq 0.524$)	A1	[0.523, 0.524] or $\frac{1}{10}(3+\sqrt{5})$ oe
	Alternative 1 (from slack position)		
	$\frac{\lambda e^2}{2l}$	M1	using formula for EPE
	$\frac{\lambda e^2}{2l} = mg(e+0.2)$	M1	equating EPE and GPE
	$20e^2 = 4(e+0.2)$	A1	correct equation
	$25e^2 - 5e - 1 = 0$ oe	A1	correct simplified quadratic
	$x = 0.2 + \frac{1\pm\sqrt{5}}{10} = 0.524$ (or $x \geq 0.524$)	A1	[0.523, 0.524] or $\frac{1}{10}(3+\sqrt{5})$ oe
	Alternative 2 (from eqm position)		
	$\frac{\lambda}{2l}(h+0.1)^2$	M1	using formula for EPE
	$\frac{1}{2} \frac{\lambda}{l}(h+0.1)^2 = mg(h+0.3)$	M1	equating EPE and GPE
	$20(h+0.1)^2 = 4(h+0.3)$	A1	correct equation
$20h^2 - 1 = 0$ oe	A1	correct simplified quadratic	
$x = 0.3 + \frac{1}{\sqrt{20}} = 0.524$ (or $x \geq 0.524$)	A1	[0.523, 0.524] or $\frac{1}{10}(3+\sqrt{5})$ oe	

Question	Answer	Marks	Partial Marks
8(ii)	Alternative 3 (using SHM equation) $0.4g - \frac{8(x-0.2)}{0.2} = 0.4\ddot{x}$ leading to	M1	deriving SHM formula in recognisable form
	$\ddot{x} + 100x = 30$ $0.4 \times 10 \times 0.2 = \frac{1}{2} \times 0.4 \times v^2 \Rightarrow v = 2$	M1	Dep on previous M. Energy consideration leading to v as string goes slack
	$x = A \cos 10t + B \sin 10t + 0.3$ or $x = R \cos(10t + \phi) + 0.3$	A1	full correct solution with $\omega = 10$ and 2 arbitrary constants
	$R \cos \phi = -0.1$ and $10R \sin \phi = 2$ and attempt to solve simultaneously	M1	using $t = 0$, $x = 0.2$ and $v = 2$ in correct equations to find A and B or R and ψ
	$R = \sqrt{A^2 + B^2} = \frac{\sqrt{5}}{10} = 0.224$ so $x = 0.524$	A1	cao
	Alternative 4 (using SHM energy equation) $0.4g - \frac{8(x-0.2)}{0.2} = 0.4\ddot{x}$ leading to	M1	deriving SHM formula in recognisable form
	$\ddot{x} + 100x = 30$ $0.4 \times 10 \times 0.2 = \frac{1}{2} \times 0.4 \times v^2 \Rightarrow v = 2$	M1	Dep on previous M. Energy consideration leading to v as string goes slack
	$\omega = 10 \Rightarrow v^2 + 100(x - 0.3)^2 = 100R^2$	A1	value of ω soi and fully correct energy equation with 3 terms and an arbitrary constant
	When $x = 0.2$, $v = 2$ so $100R^2 = 5$	M1	using $x = 0.2$ and $v = 2$ in correct equation to find A
$\therefore x = \frac{\sqrt{5}}{10} + 0.3 = 0.524$	A1	cao	
9(i)	$\cos/\sin \theta = 0.8/0.6$	B1	3:4:5 triangle implied
	$T \cos \theta = T \sin \theta + 0.2g$	M1	resolving and balancing forces vertically (trig values not nec)
	$0.8T = 0.6T + 2$	A1	correct equation with values
	$T = g = 10 \text{ N}$	A1	$T = g$ or 10

Question	Answer	Marks	Partial Marks
9(ii)	$r = 0.48$	B1	radius 0.48, can be implied or seen in (i) (or correct cancellation)
	$T \sin \theta + T \cos \theta = mr\omega^2$ or mv^2/r	M1	NII horizontally = $mr\omega^2$
	$0.6T + 0.8T = 0.2 \times 0.48 \omega^2$ or $0.2v^2/0.48$	A1	FT for correct equation
	$\omega = 12.1$ or $v = 5.80$ (or 5.81 or $\sqrt{33.6}$)	A1	correct v or ω
	Time $2\pi \div \omega = 0.520$ seconds	A1	0.52 or awrt 0.520
10(i)(a)	$-mg \sin \theta = mr\ddot{\theta}$ cwo	M1	NII tangentially. Ignore radial
	$\ddot{\theta} = -\frac{g}{r} \sin \theta \approx -\frac{g}{r} \theta$ so approx. SHM	A2	A1 for $\ddot{\theta}$ correct including signs A1 for $\sin \theta \approx \theta$ and stating SHM
	Period $2\pi \div \omega = 2.35(095)$ (seconds)	B1	awrt 2.35 or $\frac{1}{5}\pi\sqrt{14}$
	Alternative 1 (Hor & Vert)		
	$-T \sin \theta = ma \cos \theta$ and	M1	NII (\updownarrow) and (\leftrightarrow)
	$T \cos \theta - mg \sin \theta = ma \sin \theta$ cwo	A1	both fully correct
	$\ddot{\theta} = -\frac{g}{r} \sin \theta \approx -\frac{g}{r} \theta$ so approx. SHM	A1	eliminating T and using $\sin \theta \approx \theta$ and $a = r\ddot{\theta}$ and stating SHM
	Period $2\pi \div \omega = 2.35(095)$ (seconds)	B1	awrt 2.35 or $\frac{1}{5}\pi\sqrt{14}$
	Alternative 2 (x)		
	$-mg \sin \theta = m\ddot{x}$ cwo	M1	NII tangentially. Ignore radial
$\ddot{x} = -g \sin \theta \approx -g\theta = -\frac{g}{r}x$ so approx. SHM	A2	A1 for \ddot{x} correct including signs A1 for using $\sin \theta \approx \theta$ and $x = r\theta$ (or just $x \approx r \sin \theta$) and stating SHM	
Period $2\pi \div \omega = 2.35(095)$ (seconds)	B1	awrt 2.35 or $\frac{1}{5}\pi\sqrt{14}$	

Question	Answer	Marks	Partial Marks
10(i)(a)	Alternative 3 (energy) $\frac{1}{2}mv^2 + mgr - mgr \cos \theta = \text{const}$	M1	adding KE and PE (condone sign errors)
	$v = r\dot{\theta}$ (or this and $\dot{v} = r\ddot{\theta}$) used to derive DE	M1	eliminating v (and/or \dot{v})
	$mr^2\dot{\theta}\ddot{\theta} + mgr \sin \theta \dot{\theta} = 0 \Rightarrow \ddot{\theta} + \frac{g}{r}\theta \approx 0$ so approx. SHM	A1	differentiating implicitly, using $\sin \theta \approx \theta$ and rearranging and stating SHM
	Period $2\pi \div \omega = 2.35(095)$ (seconds)	B1	awrt 2.35 or $\frac{1}{5}\pi\sqrt{14}$
10(i)(b)	$\dot{\theta}^2 = \omega^2(\theta_0^2 - \theta^2)$	M1	SHM energy equation in terms of θ and $\dot{\theta}$ used
	$= \frac{10}{1.4}(0.3^2 - 0.2^2)$	A1	FT for correct equation with their ω
	$v = r\dot{\theta} = 1.4 \times 0.5976\dots$	M1	using $v = r\dot{\theta}$
	$v = 0.836(66\dots)$ or $\sqrt{70/10}$ or awrt 0.837	A1	converting to v
	Alternative 1 (x) $v^2 = \omega^2(a^2 - x^2)$	M1	SHM energy equation in terms of v and x used
	$x = 1.4\theta$ so i	M1	either $x = 0.28$ or $a = 0.42$
	$v^2 = \frac{10}{1.4}(0.42^2 - 0.28^2)$	A1	FT for correct equation with their ω
	$v = 0.836(66\dots)$ or $\sqrt{70/10}$ or awrt 0.837	A1	
	Alternative 2 (θ solution) Solution to SHM equation is $\theta = A \cos \omega t$ (+ $B \sin \omega t$) so initial conditions $\Rightarrow \theta = 0.3 \cos \omega t$	M1	particular solution, their ω
	Substituting $\omega t = \cos^{-1} \frac{2}{3}$ into $\dot{\theta} = \pm 0.3\omega \sin \omega t$	M1	Award if \sin^{-1} into $\pm 0.3\omega \cos \omega t$
	$v = r\dot{\theta} = 1.4 \times 0.5976\dots$	M1	using $v = r\dot{\theta}$
	$v = 0.836(66\dots)$ or $\sqrt{70/10}$ or awrt 0.837	A1	converting to v (must be +ve)

Question	Answer	Marks	Partial Marks
10(i)(b)	Alternative 3 (x solution) Gen sol to SHM eqn is $x = A \cos \omega t (+B \sin \omega t)$	M1	
	so initial conditions $\Rightarrow x = 0.42 \cos \omega t$	A1	using $t = 0 \Rightarrow$ both $v = 0$ and $x = 1.4 \times 0.3$
	Substituting $\omega t = \cos^{-1} \frac{2}{3}$ into $\dot{x} = \pm 0.42 \omega \sin \omega t$	M1	Award if \sin^{-1} in $\pm 0.42 \omega \cos \omega t$
	$v = 0.836(66\dots)$ or $\sqrt{70/10}$ or awrt 0.837	A1	must be +ve
10(ii)		M1	KE gained = PE lost
	$\frac{1}{2}mv^2 = mg(1.4 \cos 0.2 - 1.4 \cos 0.3)$	A1	correct equation
	$[v^2 = 0.692] \quad v = 0.832(13)$	A1	awrt 0.832
	% difference 0.544%	A1	awrt 0.544 (could be from correct calculations of $\dot{\theta}$)
11(i)	$U_y = -6$ soi	B1	\perp component of U
	$\therefore u_y = 3$	M1	$\pm 0.5 \times$ perp <u>component</u> .
	Impulse = $0.2(3 - -6) = 0.2(3 + 6)$	M1	$0.2 \times (u_y + U_y)$
	= 1.8 Ns	A1	ignore units
	perpendicular to and away from the plane	B1	
11(ii)	$u_x = 8, a_y = (-)8$	B1	both of these soi in (ii)
	$y = 3t - 4t^2$ (or $0 = 3 - 8 \times 0.5t$ or $-3 = 3 - 8t$)	M1	y -equation used, their u_y, a_y
	= 0 at $t = 0.75$ (or $t (= 2 \times 3/8) = 0.75$)	A1	obtaining $t = 0.75$ validly
	$x = 8t - 3t^2 = 4.3125$ or awrt 4.31	A1	using x -equation to obtain $4 \frac{5}{16}$ oe
11(iii)	$v_y = -3, \therefore u'_y = 1.5$	M1	$ u'_y = 0.5v_y $
	$1.5t - 4t^2 = 0$ or $0 = 1.5 - 8 \times 0.5t$ or $-1.5 = 1.5 - 8t$	M1	y -eqn., new u_y (and old a_y)
	$t = 0.375$	A1	finding new t
	Total time is $0.75 + 0.375 = 1.125$ cwo	A1	awrt 1.13 or $1 \frac{1}{8}$

Question	Answer	Marks	Partial Marks
12(i)	$M(B): 4F = 5g \times 2 \sin 60^\circ$ $M(A): 2 \times 5g \cos 30^\circ = 4 \times T \cos 30^\circ$ $M(C): 2 \times F = 2 \times T \cos 30^\circ$ $NII(\perp AB): F + T \cos 30^\circ = 5g \cos 30^\circ$ $(NII(\parallel AB): T \sin 30^\circ + N = 5g \sin 30^\circ)$ $(NII(\leftrightarrow): N \cos 30^\circ = F \sin 30^\circ)$ $(NII(\downarrow): N \sin 30^\circ + F \cos 30^\circ + T = 5g)$	M3	M1 for attempt to take moments <u>and</u> attempt to derive one other equation by NII or moments about another point M1 for deriving one useful equation M1 for deriving a second useful equation (or 2nd and 3rd if N is involved)
	$F = 12.5\sqrt{3} = 21.65 \text{ N cwo}$	A1	[21.6, 21.7] or $12.5\sqrt{3}$ oe
	$T = 25N \text{ cwo}$	A1	25 or awrt 25.0

Question	Answer	Marks	Partial Marks
12(ii)		M1	dep for attempt at moments <u>equation</u>
	One of: $M(C): 2N_B \cos 30^\circ$ $= 2N_A \cos 60^\circ + 2F_A \cos 30^\circ + 2F_B \cos 60^\circ$ $M(A): 2W + 4F_B \sin 30^\circ = 4N_B \sin 60^\circ$ $M(B): 2W = 4N_A \sin 30^\circ + 4F_A \sin 60^\circ$	A1	one correct moments <u>equation</u>
	Then completing either: Both of: $M(C)$ $NII(\leftrightarrow): N_A \sin 60^\circ$ $= F_A \sin 30^\circ + N_B \sin 30^\circ + F_B \sin 60^\circ$ or: Three of: One of $M(C)$ & $NII(\leftrightarrow)$ $M(A)$ and/or $M(B)$ $NII(//I_A): F_A + N_B = W \cos 30^\circ$ $NII(\perp I_A): N_A = F_B + W \sin 30^\circ$ $NII(\downarrow): W + F_B \cos 60^\circ$ $= N_A \cos 60^\circ + F_A \cos 30^\circ + N_B \cos 30^\circ$	M1	Dep on first M1 for other <u>equation(s)</u> to complete: <i>either</i> two equations, neither involving W ($=5g$), <i>or</i> three equations, some involving W . In each equation condone sign errors and 60/30 or sin/cos errors and calc of mg . But correct <u>forces</u> or <u>moments</u> ‘resolved’ where necessary, required for 2nd M1 . Mark the equations used or, if both not used, the 2 that give the most marks.
	$F_A = \mu_A N_A$ <u>and</u> $F_B = \mu_B N_B$	B1	Can be on diagram or in working
	$N_B(\sqrt{3} - \mu_B) = N_A(1 + \sqrt{3}\mu_A)$ $N_B(1 + \sqrt{3}\mu_B) = N_A(\sqrt{3} - \mu_A)$	M1	Dep on second M1 for simplifying to 2 sim equations in 2 eliminatable ‘unknowns’ oe elimination
	$\frac{\sqrt{3} - \mu_B}{1 + \sqrt{3}\mu_B} = \frac{1 + \sqrt{3}\mu_A}{\sqrt{3} - \mu_A}$	M1	Dep on second M1 for obtaining single equation in μ_A and μ_B only
	$\sqrt{3}(\mu_A + \mu_B) + \mu_A \mu_B = 1$	M1	Dep on second M1 for making μ_B subject of formula and using exact values of cos/sin
	$\mu_B = \frac{1 - \sqrt{3}\mu_A}{\sqrt{3} + \mu_A}$	A1	final answer in given form, cwo ($\alpha = \sqrt{3}$) Note: Form of answer given so must see full working with factorisation leading to single term in μ_B for final M1A1 Equation must have μ_A , μ_B and $\mu_A \mu_B$ terms for M1 .